ECMI Modelling Week, July 17–24, 2016, Sofia, Bulgaria

Group 8

Red Deer Import

Antigoni Kleanthous

University of Oxford, England
antigoni.kleanthous.12@ucl.ac.uk

Essi Rasimus

Tampere University of Technology, Finland
essi.rasimus@student.tut.fi

Irina Espejo

Universitat Autonoma de Barcelona, Spain
iespejomorales@gmail.com

Renald Chalayer

Universit Blaise Pascal, France
Renald.Chalayer@etudiant.univ-bpclermont.fr

Sara Battiston

Università degli studi di Milano, Italy
sara.battiston@studenti.unimi.it

Vasil Pashov

Sofia university "St. Climent Ohridski", Bulgaria
vasil.pashov1@gmail.com

Instructor: **Milana Pavić-Čolić**

University of Novi Sad, Serbia
milana.pavic@dmi.uns.ac.rs

Abstract. Hunting parks provide closed and supervised area for hunting animals. In order to gain profit such park must provide trophies for hunters. In this project we will consider hunting deers in park Vorovo, Serbia. The quality of a deer trophy depends on the age of the deer and the inbreeding. The park has a manifest called "Game Management Plan" which spans ten years and describes how the population should change in order to reach equilibrium. Since the inbreeding is hard to predict and importing new deers is very expensive a proper model should be made in order to know when and how many deers should be imported.

8.1 Introduction

In 8.2 we analyze the current park strategy by fitting it with polynomials. In 8.3 we propose a new strategy and then in 8.4 and 8.5 we show how models for the newborns and the older deer should be derived, and in 8.6 we show that the new model that we suggest works.

The next part of the project deals with the inbreeding. For that we have implemented a Monte Carlo simulation based algorithm, which shows how the population evolves with and without importing new deers.

8.2 Current Strategy

The hunting park has provided us data corresponding to the optimal number of deers they want to achieve, the current strategy they are following and the actual number of deers observed each year. We approximate their current strategy using polynomials of different degrees to control the development of their strategy and exploiting as initial data the actual numbers of deers taken from the tables provided by the park. We use a cubic and a fifth degree polynomial each with its own limitations.
The cubic degree polynomial is a worse approximation of the strategy but it allows more freedom during the evolution of the population (as we are only fixing 3 points) while, on the other hand, a fifth order approximation fits better the data but with less freedom. We use the following polynomial for the cubic approximation:

$$y = \frac{1-y_0}{400}x^3 + \frac{23y_0-143}{400}x^2 + \frac{133-17y_0}{40}x + y_0, \tag{8.1}$$

where x is the age of the deers, y_0 the number of newborns on some particular year and y is the number of deers of age x. The polynomial approximation of degree five is given by

$$\begin{aligned} y \quad = \quad & \frac{132-7y_0}{25200}x^5 + \frac{-1457+77y_0}{8400}x^4 + \frac{53208-2863y_0}{25200}x^3 \\ & + \frac{-96253+5383y_0}{8400}x^2 + \frac{61779-3871y_0}{2520}x + y_0. \end{aligned} \tag{8.2}$$

We present the evolution of the two polynomial approximations over six years with initial data taken from the real data observed in the hunting season 2009-2010. The results for the cubic approximation can be seen in Figure 8.1 and for the fifth order polynomial approximation in Figure 8.2. The blue curves represent our approximation while the red curves the current strategy.

We also plot the current strategy followed against the optimal strategy for 10 years. We can see that their current strategy does not result in the optimal number of deers per age. We conclude that a better model of the current population along with a new strategy should be made in order to achieve that optimal number.

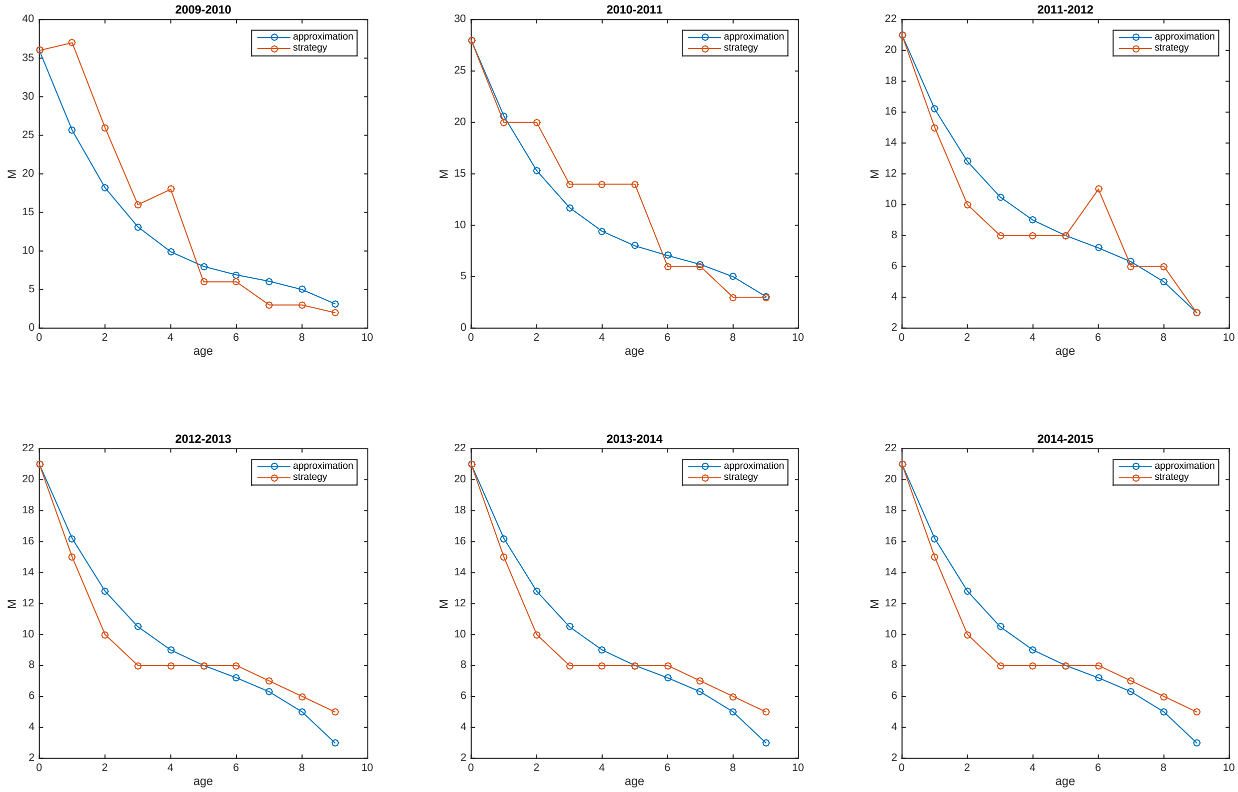


Figure 8.1: Evolution of the deer population per age for six years using the cubic polynomial approximation.

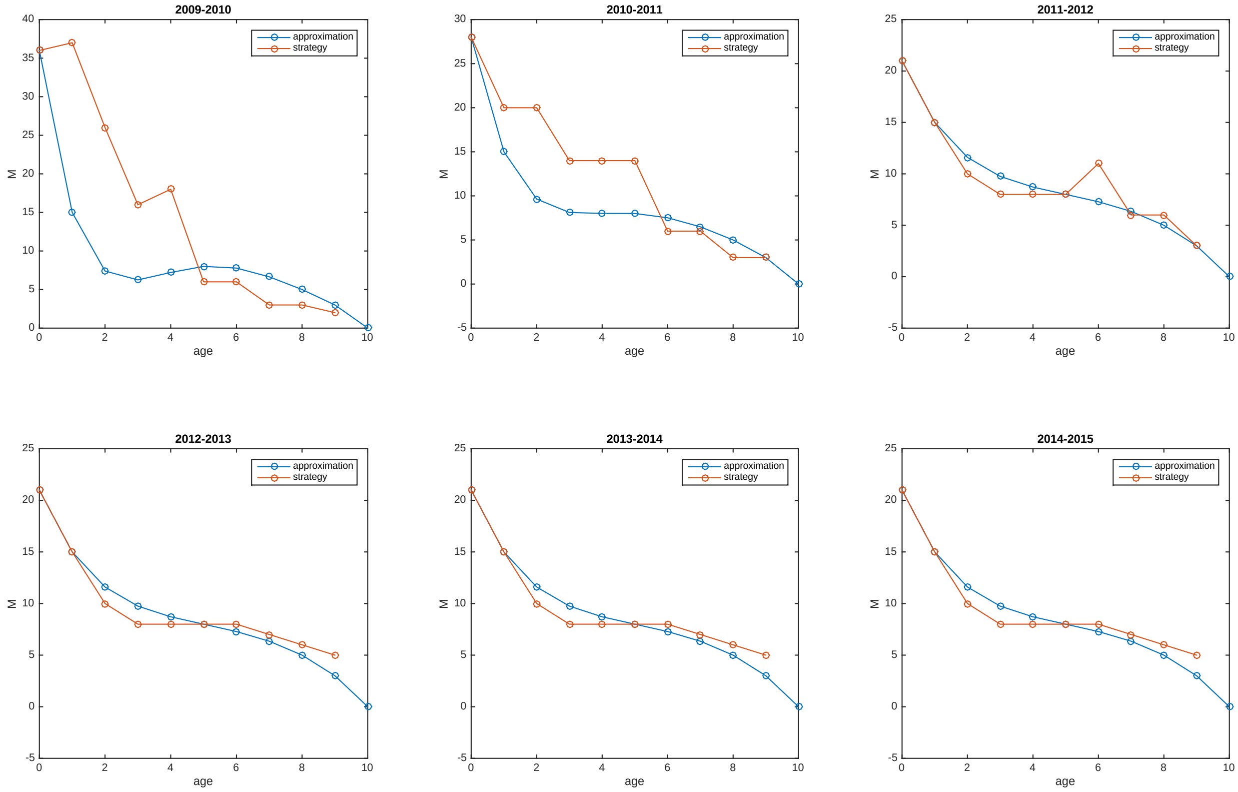


Figure 8.2: Evolution of the deer population per age for six years using the fifth order polynomial approximation.

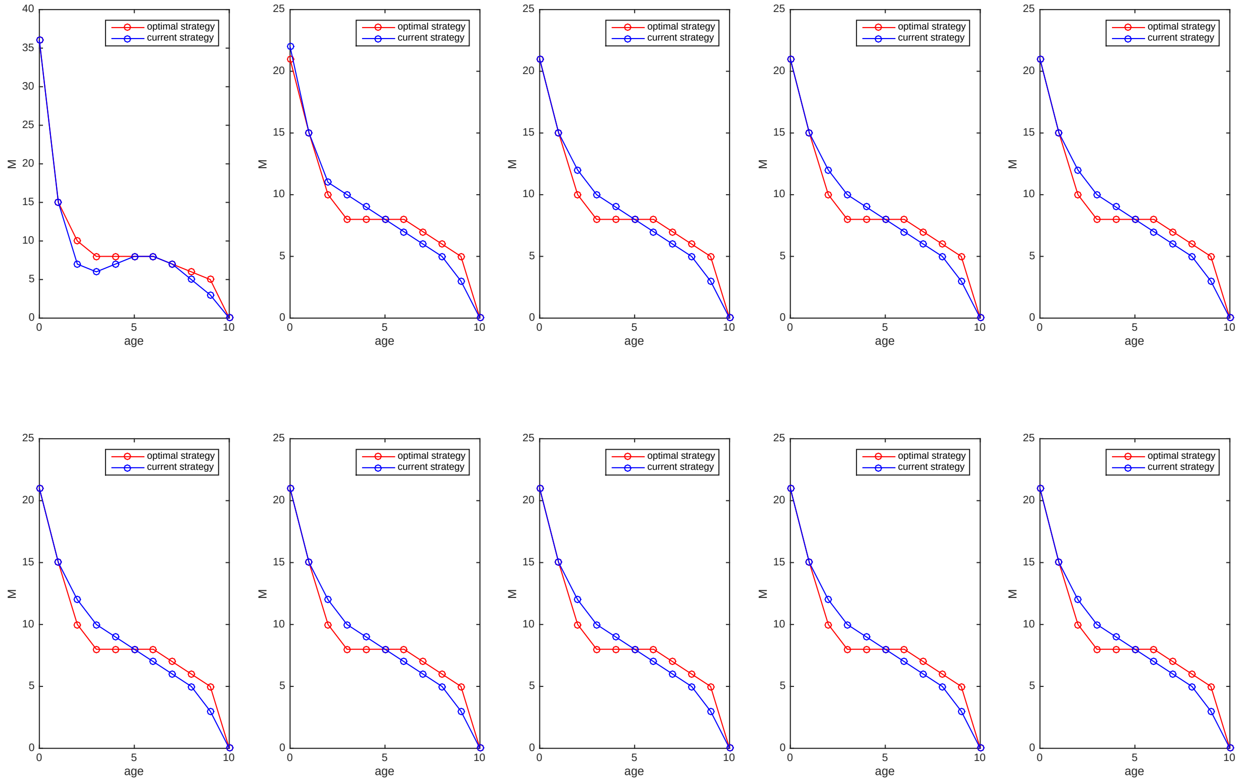


Figure 8.3: Optimal Strategy against the Current Strategy for 10 years.

8.3 Discrete Aged-Structured Population Model

To derive a model for the Red Deer population in the park we divide it into 11 age classes: newborns (0-year-olds), 1 year old deers, 2 years old deers, etc up to 10 years old deers. An optimal number of species is determined by the park, taking into account peace, environmental, climate and vegetation conditions. This optimal population is described in the table in Figure 8.4.

		0	1	2	3	4	5	6	7	8	9	10
Number of individuals before hunting current year	M	21	15	10	8	8	8	8	7	6	5	
	F	21	15	10	8	8	8	8	7	6	5	
Number of individuals before hunting following year	M	21	15	10	8	8	8	8	7	6	5	
	F	21	15	10	8	8	8	8	7	6	5	

Figure 8.4: Optimal population desired by the park.

Our proposition for the new strategy is to calculate the survival rate; that is how many deers will survive from one year to the next. Let p_i be the probability of an individual at age i to survive to age $i+1$, for $i = 0, ..., 9$. The model is based on the fact that during a year any deer that survive within a class i, age by a year and therefore move into the class $i+1$. A schematic representation of the age classes along with probabilities of survival is given in Figure 8.5. Our goal is to find the probability of survival p_i in order to reach the optimal deer population in the park in less than 10 years, and so that we can maintain this optimal deer population in the years. Such discrete population models depend strongly on the number of newborns each year and the probability of survival of each age class.

$$0 \xrightarrow{p_0} 1 \xrightarrow{p_1} 2 \xrightarrow{p_2} 3 \xrightarrow{p_3} 4 \xrightarrow{p_4} 5 \xrightarrow{p_5} 6 \xrightarrow{p_6} 7 \xrightarrow{p_7} 8 \xrightarrow{p_8} 9 \xrightarrow{p_9} 10$$

Figure 8.5: Schematic representation of age classes in the discrete population model along with probabilities of survival from one age class to the next.

8.4 Modelling of number of newborns

In order to reach our goal, we have to modify the probability p_0 in the years until we reach the optimal population.

When the dataset was analysed we noticed that the modelling of the number of newborns was inaccurate. The National Park used the number of female individuals over than 2 years old to determine the number of newborns. The real data shows that the estimated number of newborns was too small every year. When the game management plan was made using this model the optimal stage was unreachable.

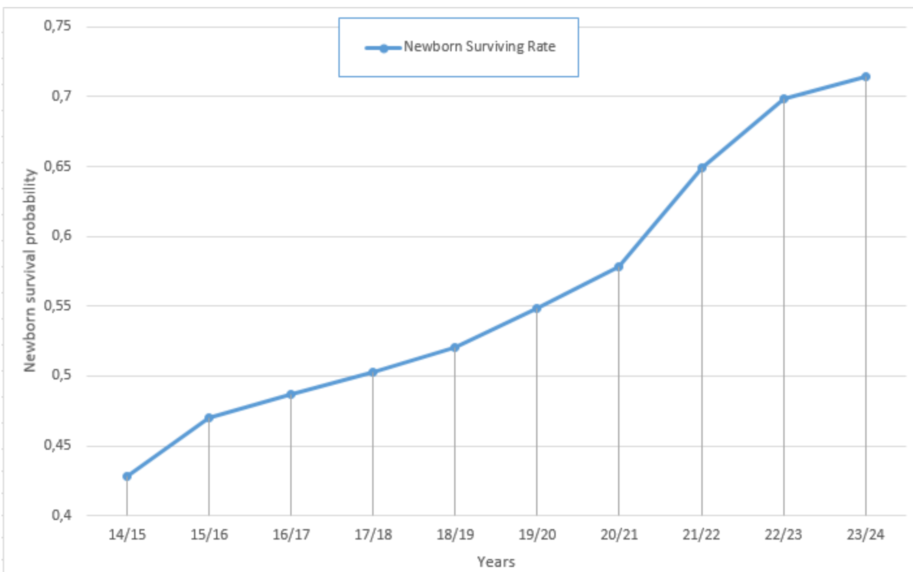
We used the real data to find the most accurate way to model the number of newborns. We scanned the relation between the number of newborns and multiple groups from the population. From the relations we determined the means and the standard errors of the mean. This confirmed that the former way to model the number of newborns was inexact. The most accurate way to estimate the number of newborns is to use the total number of population after hunting on the previous year. Our proposition to estimate the number of newborns is by the following formula

$$(\text{Newborns})_{t,beforehunting} = \frac{21}{150}(\text{Total Population})_{t-1,afterhunting}.$$

Because the number of newborns is not constant during the years, the probability p_0 is going to vary through the years. After the optimal stage is reached the probability p_0 will stay constant. The probability p_0 in the year t is given by

$$p_0 = \frac{15}{Newborns_{t,beforehunting}}.$$

If we start the model with the real data of the season 2014-2015, we obtain the following evolution of the probability p_0



From the graph we see that the probability for newborns to survive will increase until the optimal stage is reached. The probability will increase because in the initial stage there are too many newborns and during the years the number of newborns will decrease.

8.5 Modelling of number of the older deers

We can reach the optimal population by choosing constant values for the probabilities p_i for $i = 1, ..., 9$. In order to determine these probabilities, we propose a linear model based on the number of deer in each age class in the optimal situation.

Let us explain how we obtain p_1, then we will use the same method to find $(p_i)_{i=2...9}$. We recall the optimal population is given by

		0	1	2	3	4	5	6	7	8	9	10
Number of individuals before hunting current year	M	21	15	10	8	8	8	8	7	6	5	
	F	21	15	10	8	8	8	8	7	6	5	
Number of individuals before hunting following year	M	21	15	10	8	8	8	8	7	6	5	
	F	21	15	10	8	8	8	8	7	6	5	

If we have 15 one-year-old deer in the year t, and we want 10 two-years-old deer in the year $t + 1$, we have to remove $\frac{10}{15}$ per cent of the one-year-old deers in order to keep the equilibrium. If we have more than 15 one-year-old deers in the year t (like in the season 2014/2015 for example), we will also reach the equilibrium in a number of years by removing $\frac{10}{15}$ per cent of the one-year-old deer. In conclusion, we define $p_1 = \frac{10}{15}$. Using the same reasoning, we define

$$p_2 = \frac{8}{10}, \quad p_3 = p_4 = p_5 = 1, \quad p_6 = \frac{7}{8}, \quad p_7 = \frac{6}{7}, \quad p_8 = \frac{5}{6}, \quad p_9 = 0.$$

8.6 The Game Management Plan

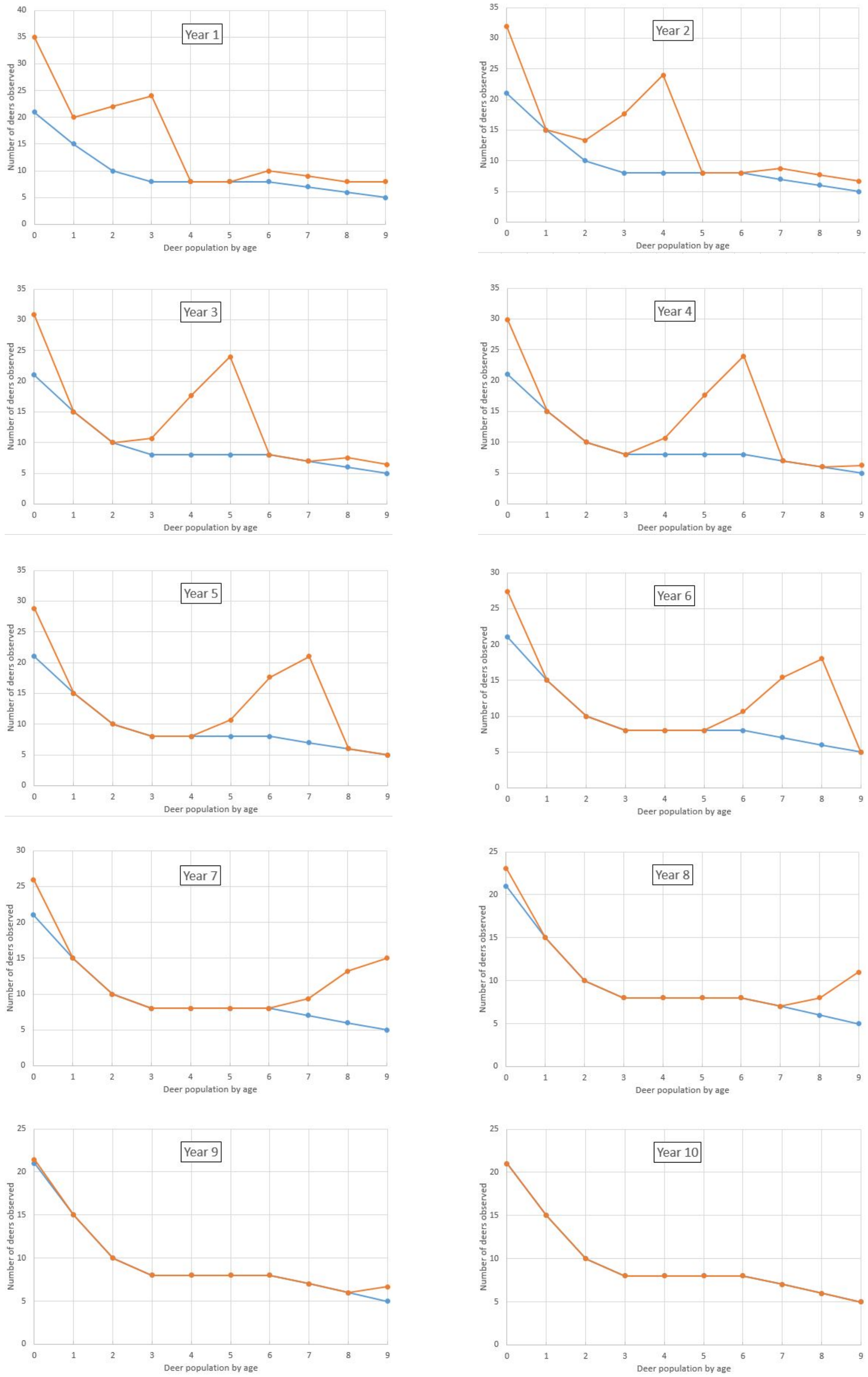


Figure 8.6: The game management plan for male deer starting from the year 2014-2015. Blue line refers to the optimal stage and orange line is the population each year. Year 1 corresponds to 2014-2015 and Year 10 to 2023-2024.

In Figure 8.6 we present the game management plan for male deers in the years 2014-2024. The real data from the season 2014-2015 was used as the initial number of deers. The real age structure is presented on the orange line. Every graph presents also the optimal number of deers on the blue line. The graphs were obtained using the discrete age-structured population model along with the computed probabilities derived in the earlier sections.

The graphs show that the optimal stage will be reached in 10 years time frame regardless of the initial conditions. After the 10th year of the game management plan the optimal stage will remain. In addition during these 10 years there will be more than optimal number of deer in the age of 8 or more. These deer have the most valuable trophies during hunting so the National Park will receive more profits during these years.

The graphs only present population of male deers because a similar behaviour for female deers it's shown and we are more interested in the number of old male deers. The final game management plans for the years 2014-2024 are presented in the appendix .1. The plans include the numbers of deers for both genders. Also the plans contain estimated numbers of deer to hunt and loss every year.

8.7 Inbreeding

In this section we are going to deal with the inbreeding phenomena, precisely with inbreeding population depression. Inbreeding depression is the decrease in the population of individuals of a species where endogamy takes place, it is related to the inheritance of some recessive alleles that provoke malformations and diseases in the new generations . In our case, non mortal malformations such as malformations in the trophy are crucial for the well run of the hunting business of the Vorovo area.

8.7.1 Assumptions

Due to the complexity that the problem presents, the following assumptions regarding to different aspects of red deer life were introduced

Genetics

- The degeneration has a recessive behavior.
- The genes follow Mendelian Inheritance.
- The deer population is classified in three genetic categories as in Table 8.1.

Healthy [0]	Carrier [1]	Malformed [2]
AA	Aa	aa

Table 8.1: Table that sums up the three genetic categories.

Survival

- Healthy young deers are not hunted in any season.
  The acceptance of this assumption is the only way that we can extract some informations about genetics from the given dataset. Consequently, this simplification sets us in a worst case scenario where young healthy deer dies only due to other unknown reasons.
- Birth and death are probabilistic events fitted by the data thus they depend on sex, age and genetics.
- One male can mate with more than one female.

8.7.2 Computational Model

Introduction

Solving the problem analytically might be hard for just one week, for that reason we decided for a computational approach. Since finding equations for the evolution of genetics is difficult and therefore we cannot make simulations from them, we are going to perform a Montecarlo simulation.

We are interested in the situation from 10 years on, starting from the actual initial conditions for the deer population. For this reason, in every trial of the Montecarlo method the initial conditions will not vary. The stochastic events will be death and reproduction and they take place every year. (See sections below for a full explanation.)

Finally, with the results of all the trials we averaged and obtained the distribution of every deer category over every year from now to 10 years on.

How it works

Every deer is an item of an array that contains all the population, a single deer will have 3 attributes:

- Age: [0,10] is the age in Years.
- Gender: [0,1] where 0 is Male and 1 is Female.
- Genetics: [0,2] where 0 is Healthy, 1 is Carrier, and 2 is Malformed. (therefore the index is equal to the number of wrong genes).

Individuals which have 00 genes will be healthy, 01 or 10 healthy carriers and 11 ill carriers. The data will be saved in a 3-dimensional array for a total of 80 elements (total of deers).

Regarding the evolution of the time in the program, we start simply by aging one year all the deers and eliminating all the deers with age grater than 10. In addition to the natural passing of time, other events of stochastic nature can occour during a year:

- Generation of Newborns
- Selective Hunting season
- Trophy Hunting season
- Import of healthy deer

By the end of the year, one can count the number of alive deers in each category (age, sex, genetics). The whole procedure is repeated for 10 years, so that we can evaluate if the final stage is a desired one or not.
Then, we repeat the 10-year calculations and we take the average for every year for the number of deers of the category chosen. See the diagram 8.7 for more clarity.

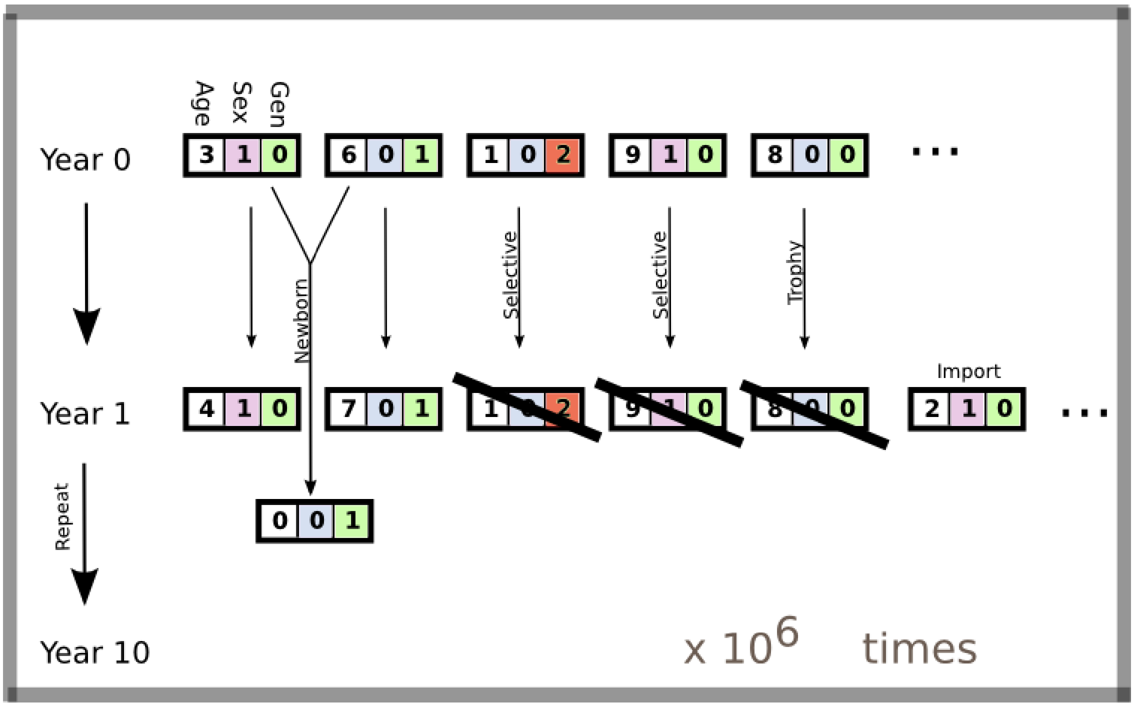


Figure 8.7: Diagram that explains the structure of the program.

Initial conditions and other parameters

In the previous section we analysed in detail how the program works but the parameters that appear in it were left unexplained. In this section we will show what they represent and how they are computed.

- **Generation of the newborns**
  For simplification we have implemented the following (which could be addressed at a later point).
  Males reproduce after turning 5 y.o. and females after turning 2 y.o..Once they reach their sexual maturity there is no practical difference between the individuals with respect to their age.
  The health/illness of individuals has no effect on their ability to reproduce, that is, they reproduce with each other with the same probability independently of their illness. The last gives the following equations for the genetic expected values of the newborns:

$$E(I) \propto I_M \times I_f + \frac{1}{2}C_M \times I_f + \frac{1}{2}C_f \times I_m + \frac{1}{4}C_m \times C_f \tag{8.3}$$

$$E(C) \propto \frac{1}{2}C_M \times I_f + \frac{1}{2}I_M \times C_f + \frac{1}{2}C_f \times C_m + \frac{1}{2}C_M \times H_f + \frac{1}{2}H_M \times C_f \tag{8.4}$$

$$E(H) \propto H_M \times H_f + \frac{1}{2}C_M \times H_f + \frac{1}{2}C_f \times H_m + \frac{1}{4}C_m \times C_f \tag{8.5}$$

Where I represent the ill individuals, C the carriers individuals, H the healthy individuals and the subindexes represent the sex, f female and m male. Those modelling equations arise from the well-know mendeleian diagrams for mating(See figure 8.8) and through them we can fully define how the new generation has to be each year. More precisely, we are going to assign to each new individual its genetic traits, but from an higher level the expected values have to match the equations.

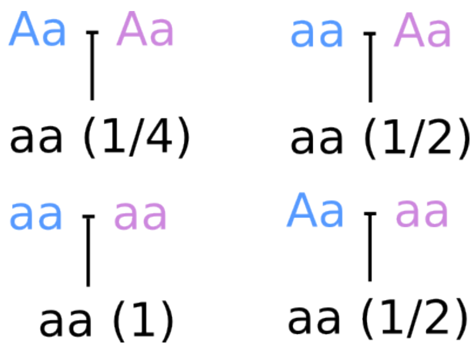


Figure 8.8: Diagram representing the reasoning behind equation 8.3, purple females and blue males.

- **Selective hunting**
  By selective hunting we mean the condition for which hunters eliminate the malformed deers that they can see. As they are not perfect hunters, the process is regulated by some probabilities which depend on the age, gender and genetics of the population observed. Thus malformed deers have higher chances of being hunted than healthy or carriers ones in the current season. Furthermore, one would expect that it is easier to hunt a young individual than an older one. However, we have to make sure somehow that by the trophy age all the malformed deers have been killed. The inference of these probabilities is empirical, coming from the data provided by the park. For the healthy elements it is clear that we should impose that the probability of being selectively hunted is zero,$p_{selec_hunt} = 0$, as under our assumptions hunters do not kill healthy individuals.
  For the malformed deers, the probability becomes $p_{selec_hunt} = \frac{H_i}{N_i}$, where H_i is the number of hunted deers from the category i (age, sex, genes) and N_i is the number of total deers in this category. Applying this, we can summarize all the probabilities used in our simulation in the table below.

- **Trophy hunting**
  In the trophy season only grown adult males are hunted, therefore the probability for the young of being hunted is zero and so it is for all females. We have also to make sure that by the age of 8-9 all the trophies are hunted, so the probability rises to 1.

  Clearly in neither selective nor trophy hunting exists a difference between being a carrier or a healthy deer, as hunters cannot differentiate them.

- **Import**

Table 8.2: Table presenting the selective hunting probabilities for each year and deer category.

	Males			Females		
Genetics	0	1	2	0	1	2
Year 0	0	0	0.706	0	0	0.706
Year 1	0	0	0.765	0	0	0.765
Year 2	0	0	0.4	0	0	0.4
Year 3	0	0	0.571	0	0	0.524
Year 4	0	0	0.2	0	0	0.381
Year 5	0	0	0.4	0	0	0.286
Year 6	0	0	0.2	0	0	0.143
Year 7	0	0	1	0	0	1
Year 8	0	0	1	0	0	1
Year 9	0	0	1	0	0	1
Year 10	0	0	1	0	0	1

Table 8.3: Table presenting the trophy hunting probabilities for each year and deer category.

	Males			Females		
Genetics	0	1	2	0	1	2
Year 0	0	0	0	0	0	0
Year 1	0	0	0	0	0	0
Year 2	0	0	0	0	0	0
Year 3	0	0	0	0	0	0
Year 4	0	0	0	0	0	0
Year 5	0	0	0	0	0	0
Year 6	0	0	0	0	0	0
Year 7	0.44	0.44	0	0	0	0
Year 8	1	1	0	0	0	0
Year 9	1	1	0	0	0	0
Year 10	1	1	0	0	0	0

In order to find the optimal import demographic strategy, simulating many times over 10 years of genetic evolution with a Montecarlo method might have been time consuming and we had not much time. So, since we wanted to optimize the import, we had to impose some constraints also to prevent for example from reaching trivial optimal solutions such as killing all the population and replacing by healthy one which is obviously not desirable. Moreover these constraints should have been related to the number of original deers, the amount of money the park can spend and some boundary for the genetic variability for a certain number of years from now. Deeper examination of those aspects could become an interesting further work.

Now we decide to impose some simple constraints, propose an import solution and evaluate how the model works. We assume then:

- Newborns cannot be imported because of breastfeeding
- Females should be older than 2 and males older than 5. This way we guarantee that females are fertile and males are young enough to compete with other males for reproduction.
- Benefit = price $\times$ number of tropheys
- Budget spent on import each year ≤ 0.7 times the benefit

8.7.3 Results

The results obtained during the Modelling Week are presented in this section. With the method explained in the previous sections we can extract the time evolution probability distribution for every deer group in our model. Nevertheless, the groups that give us more significant information are the malformed newborns which are a measure of the genetic quality of the population, and the trophies male deers which are strictly related to the park's intake.
Furthermore, we compared simulations which do not take into account any importation of new deers against simulations with deers imported from the outside, where we decided to import a ratio of 3 female and 3 male deer per year.
Regarding to the malformed newborns we obtained:

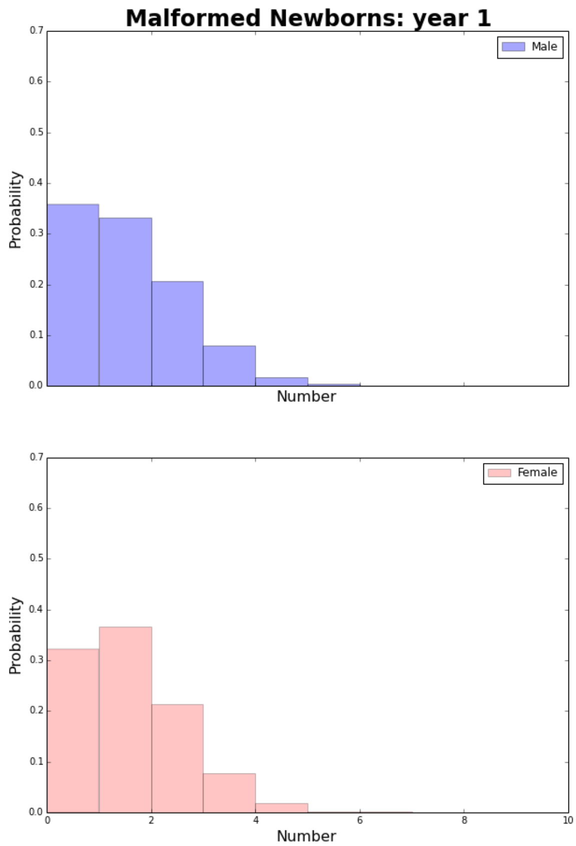


Figure 8.9: 1year malformed newbonrs

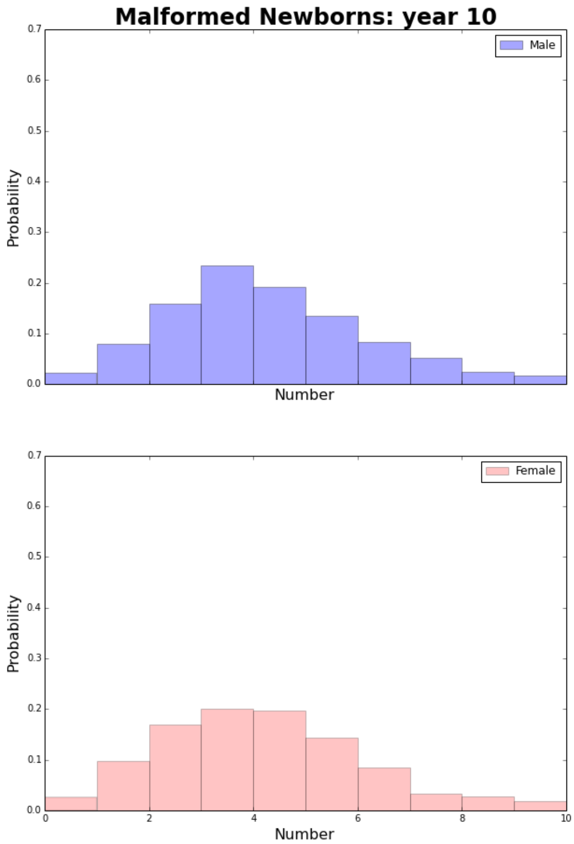


Figure 8.10: 1year malformed newbonrs

And with the import:

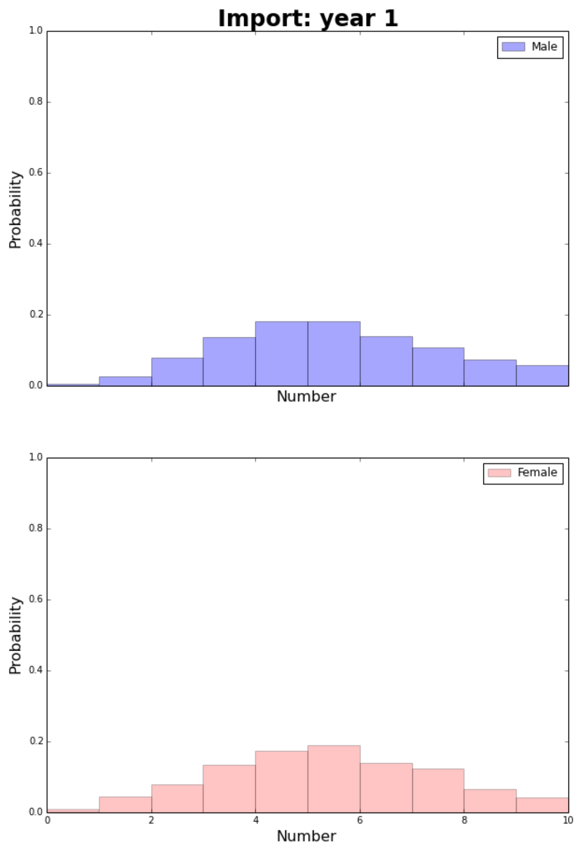


Figure 8.11: 1year malformed newbonrs

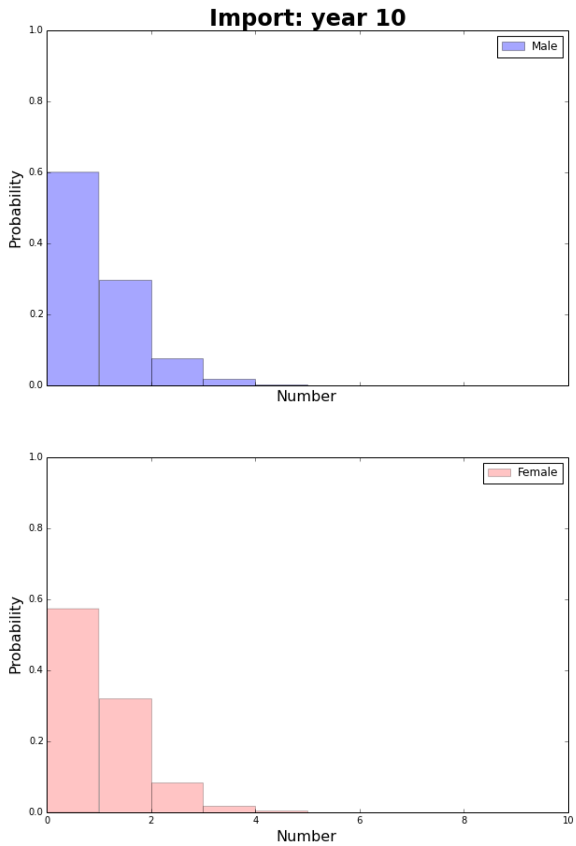


Figure 8.12: 1year malformed newbonrs

On the other hand, the probability distribution for the trophies was:

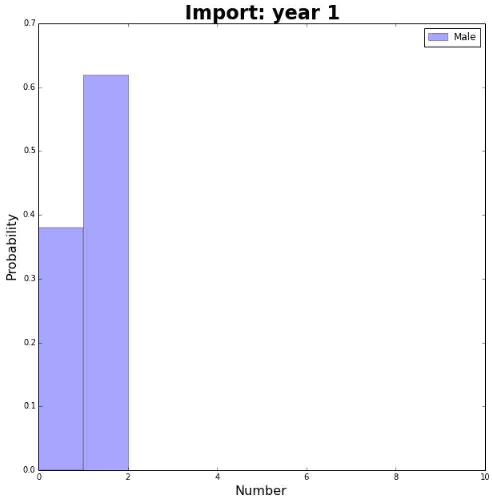


Figure 8.13

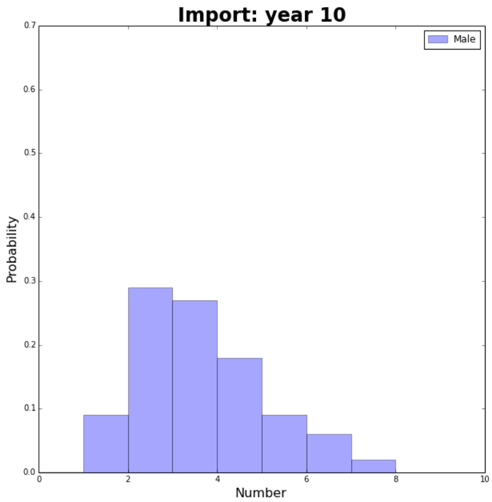


Figure 8.14

8.8 Conclusions

We have proven that the current strategy in the park is not optimal and proposed a new strategy which we have shown to be better and to converge to the optimal expected numbers for the park in the desired timespan.

The genes of the deers are also important and should be taken into account by the park to optimize their income. The simple assumptions we have made suffice to make a basic genetic model through which we have shown that some import

is necessary to prevent the malformations from happening and the female/male import ratio suggested per year appears to be a satisfying solution, though this could be expensive for the park.
An optimal strategy should be found in a further work since it requires more complex knowledge of genetics and more time to perform longer simulation.

.1 The Game Management Plans 2014-2024

		2014-2015											2015-2016									
Category		0	1	2	3	4	5	6	7	8	9	10	0	1	2	3	4	5	6	7	8	9
Optimal Number of Species	M	15	10	8	8	8	8	7	6	5			15	10	8	8	8	8	7	6	5	
	F	15	10	8	8	8	8	7	6	5			15	10	8	8	8	8	7	6	5	
Number of individuals 31 March	M	20	22	24	8	8	10	9	8	8	0		15	13	18	24	8	8	9	8	7	0
	F	20	20	20	14	14	10	12	9	10	4		15	13	16	20	14	14	9	10	8	0
Birthday 1st April	M		20	22	24	8	8	10	9	8	8	0		15	13	18	24	8	8	9	8	7
	F		20	20	20	14	14	10	12	9	10	4		15	13	16	20	14	14	9	10	8
Expected Newborns	M	35											32									
	F	35											32									
Number of individuals before hunting	M	35	20	22	24	8	8	10	9	8	8	0	32	15	13	18	24	8	8	9	8	7
	F	35	20	20	20	14	14	10	12	9	10	4	32	15	13	16	20	14	14	9	10	8
Expected harvest and losses	M	20	7	4	0	0	0	1	1	1	8	0	17	5	3	0	0	0	1	1	1	7
	F	20	7	4	0	0	0	1	2	2	10	4	17	5	3	0	0	0	2	1	2	8
Expected number of species at the end of hunting year	M	15	13	18	24	8	8	9	8	7	0	0	15	10	11	18	24	8	7	8	6	0
	F	15	13	16	20	14	14	9	10	8	0	0	15	10	11	16	20	14	12	8	9	0
Relation with respect to the optimal number	M	0	3	10	16	0	0	2	2	2	0	0	0	0	3	10	16	0	0	2	1	0
	F	0	3	8	12	6	6	2	4	3	0	0	0	0	3	8	12	6	5	2	4	0

2016-2017										2017-2018										2018-2019									
0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9
15	10	8	8	8	8	7	6	5		15	10	8	8	8	8	7	6	5		15	10	8	8	8	8	7	6	5	
15	10	8	8	8	8	7	6	5		15	10	8	8	8	8	7	6	5		15	10	8	8	8	8	7	6	5	
15	10	11	18	24	8	7	8	6	0	15	10	8	11	18	24	7	6	6	0	15	10	8	8	11	18	21	6	5	0
15	10	11	16	20	14	12	8	9	0	15	10	8	11	16	20	12	11	6	0	15	10	8	8	11	16	18	11	9	0
	15	10	11	18	24	8	7	8	6		15	10	8	11	18	24	7	6	6		15	10	8	8	11	18	21	6	5
	15	10	11	16	20	14	12	8	9		15	10	8	11	16	20	12	11	6		15	10	8	8	11	16	18	11	9
31										30										29									
31										30										29									
31	15	10	11	18	24	8	7	8	6	30	15	10	8	11	18	24	7	6	6	29	15	10	8	8	11	18	21	6	5
31	15	10	11	16	20	14	12	8	9	30	15	10	8	11	16	20	12	11	6	29	15	10	8	8	11	16	18	11	9
16	5	2	0	0	0	1	1	1	6	15	5	2	0	0	0	3	1	1	6	14	5	2	0	0	0	2	3	1	5
16	5	2	0	0	0	2	2	1	9	15	5	2	0	0	0	3	2	2	6	14	5	2	0	0	0	2	3	2	9
15	10	8	11	18	24	7	6	6	0	15	10	8	8	11	18	21	6	5	0	15	10	8	8	8	11	15	18	5	0
15	10	8	11	16	20	12	11	6	0	15	10	8	8	11	16	18	11	9	0	15	10	8	8	8	11	14	15	9	0
0	0	0	3	10	16	0	0	1	0	0	0	0	0	3	10	14	0	0	0	0	0	0	0	0	3	8	12	0	0
0	0	0	3	8	12	5	5	1	0	0	0	0	0	3	8	11	5	4	0	0	0	0	0	0	3	7	9	4	0

2019-2020										2020-2021										2021-2022									
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
15	10	8	8	8	8	7	6	5		15	10	8	8	8	8	7	6	5		15	10	8	8	8	8	7	6	5	
15	10	8	8	8	8	7	6	5		15	10	8	8	8	8	7	6	5		15	10	8	8	8	8	7	6	5	
15	10	8	8	8	11	15	18	5	0	15	10	8	8	8	8	9	13	15	0	15	10	8	8	8	8	7	8	11	0
15	10	8	8	8	11	14	15	9	0	15	10	8	8	8	8	9	12	13	0	15	10	8	8	8	8	7	8	10	0
	15	10	8	8	8	11	15	18	5		15	10	8	8	8	8	9	13	15		15	10	8	8	8	8	7	8	11
	15	10	8	8	8	11	14	15	9		15	10	8	8	8	8	9	12	13		15	10	8	8	8	8	7	8	10
27										26										23									
27										26										23									
27	15	10	8	8	8	11	15	18	5	26	15	10	8	8	8	8	9	13	15	23	15	10	8	8	8	8	7	8	11
27	15	10	8	8	8	11	14	15	9	26	15	10	8	8	8	8	9	12	13	23	15	10	8	8	8	8	7	8	10
12	5	2	0	0	0	1	2	3	5	11	5	2	0	0	0	1	1	2	15	8	5	2	0	0	0	1	1	1	11
12	5	2	0	0	0	1	2	3	9	11	5	2	0	0	0	1	1	2	13	8	5	2	0	0	0	1	1	1	10
15	10	8	8	8	8	9	13	15	0	15	10	8	8	8	8	7	8	11	0	15	10	8	8	8	8	7	6	7	0
15	10	8	8	8	8	9	12	13	0	15	10	8	8	8	8	7	8	10	0	15	10	8	8	8	8	7	6	7	0
0	0	0	0	0	0	2	7	10	0	0	0	0	0	0	0	0	2	6	0	0	0	0	0	0	0	0	0	2	0
0	0	0	0	0	0	2	6	8	0	0	0	0	0	0	0	0	2	5	0	0	0	0	0	0	0	0	0	2	0

2022-2023										2023-2024									
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
15	10	8	8	8	8	7	6	5		15	10	8	8	8	8	7	6	5	
15	10	8	8	8	8	7	6	5		15	10	8	8	8	8	7	6	5	
15	10	8	8	8	8	7	6	7	0	15	10	8	8	8	8	7	6	5	0
15	10	8	8	8	8	7	6	7	0	15	10	8	8	8	8	7	6	5	0
	15	10	8	8	8	8	7	6	7		15	10	8	8	8	8	7	6	5
	15	10	8	8	8	8	7	6	7		15	10	8	8	8	8	7	6	5
21										21									
21										21									
21	15	10	8	8	8	8	7	6	7	21	15	10	8	8	8	8	7	6	5
21	15	10	8	8	8	8	7	6	7	21	15	10	8	8	8	8	7	6	5
6	5	2	0	0	0	1	1	1	7	6	5	2	0	0	0	1	1	1	5
6	5	2	0	0	0	1	1	1	7	6	5	2	0	0	0	1	1	1	5
15	10	8	8	8	8	7	6	5	0	15	10	8	8	8	8	7	6	5	0
15	10	8	8	8	8	7	6	5	0	15	10	8	8	8	8	7	6	5	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Bibliography