Denoising of Poissonian Images

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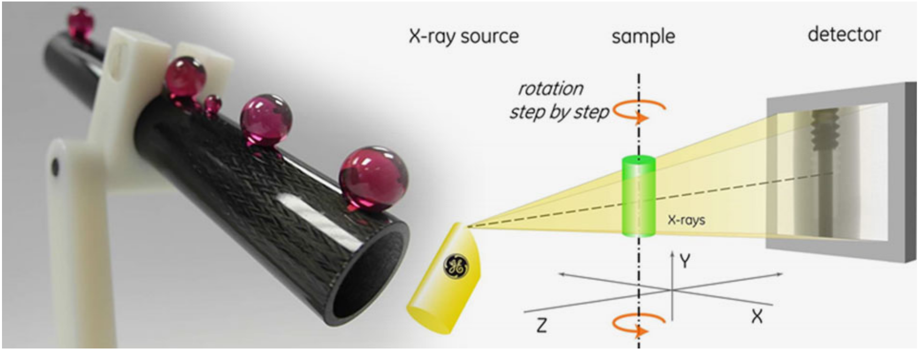
Motivation

Figure : Industrial CT scanning



In CT scanning, the 2D radiographic images are obtained by counting photons, that hit the detector surface → Poisson noise occurs and dominates the noise distribution.

Mathematical Formulation

- $\bar{u} \in [0, \nu]^n$ – original gray-scale image of intensity ν.
- $H : [0, +\infty)^{n \times n}$ – Gaussian blur operator (smoother).
- $f \in \mathbb{R}^n : \quad (f_i \sim \mathcal{P}((Hx)_i))_1^n$ – observed (blurred and noisy) image.

Goal: Given f, reconstruct $\bar{u}$ (Imaging inverse problem).

- Penalized optimization:

$$\boxed{\operatorname*{argmin}_u \Psi(u) + \lambda F(u, f), \qquad \lambda \geq 0}$$

- Constraint optimization:

$$\boxed{\operatorname*{argmin}_u \Psi(u) \quad \text{subject to} \quad F(u, f) \leq \tau, \qquad \tau \geq 0}$$

- Ψ – regularization term (convex, proper, lsc.)
- F – data fidelity term (convex, proper, lsc.)

Constraint TV optimization

$\|\nabla u\|_{2,1}$ – Total Variation (TV) semi-norm [Rudin, Osher, Fatemi 1992] .

- Anscombe constrained problem [Chaux et al. 2009, Dupé et al. 2009] :

$$\boxed{\underset{u\in[0,+\infty)^n}{\operatorname{argmin}} \ \|\nabla u\|_{2,1} \quad \text{subject to} \quad \|T(Hu) - T(f)\|_2^2 \le \tau_A}$$

$$T\colon [0,+\infty)^n \to (0,+\infty)^n\colon (v_i)_{1\le i\le n} \mapsto 2\left(\sqrt{v_i + \frac{3}{8}}\right)_{1\le i\le n}$$

- I-div constrained problem [Carlavan, Blanc-Féraud 2012, Teuber, Steidl, Chan 2013]:

$$\boxed{\underset{u\in[0,+\infty)^n}{\operatorname{argmin}} \ \|\nabla u\|_{2,1} \quad \text{subject to} \quad D(f, Hu) \le \tau_I,}$$

$$D(f, Hu) := \begin{cases} \langle \mathbf{1}_n, f \log \frac{f}{Hu} - f + Hu\rangle & \text{if } Hu > 0, \\ +\infty & \text{otherwise.} \end{cases}$$

Solution [Harizanov, Pesquet, Steidl 2013] & [Teuber, Steidl, Chan 2013]

- Algorithm: **P**rimal-**D**ual **H**ybrid **G**radient [Chambolle, Pock 2011].
- Constraints: $\tau_A = n$, $\tau_I = n/2$.
- Image quality measures: $\mathrm{PSNR} = 10\log_{10} \frac{|\max \bar{u} - \min \bar{u}|^2}{\frac{1}{n}\|u-\bar{u}\|_2^2}$, $\mathrm{MAE} = \frac{1}{n}\|\bar{u} - u\|_1$.

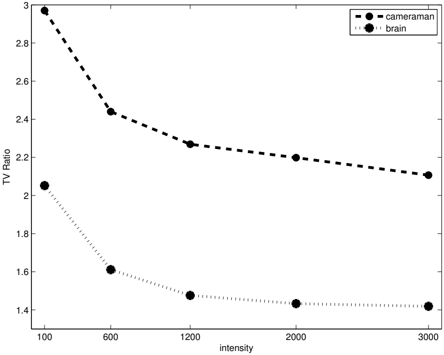


Figure : The ratio $TV(\bar{u})/TV(u_A)$ as a function of ν.

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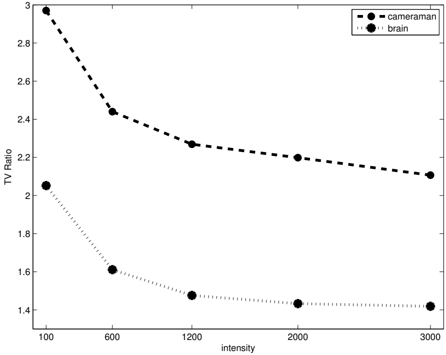

Figure : The ratio $TV(\bar{u})/TV(u_A)$ as a function of ν.

$\Rightarrow$ the constraint sets $C_A = \{u : \|T(Hu) - T(f)\|_2^2 \le \tau_A\}$, $C_I = \{u : D(f, Hu) \le \tau_I\}$ need to be restricted.

Restriction via decreasing τ

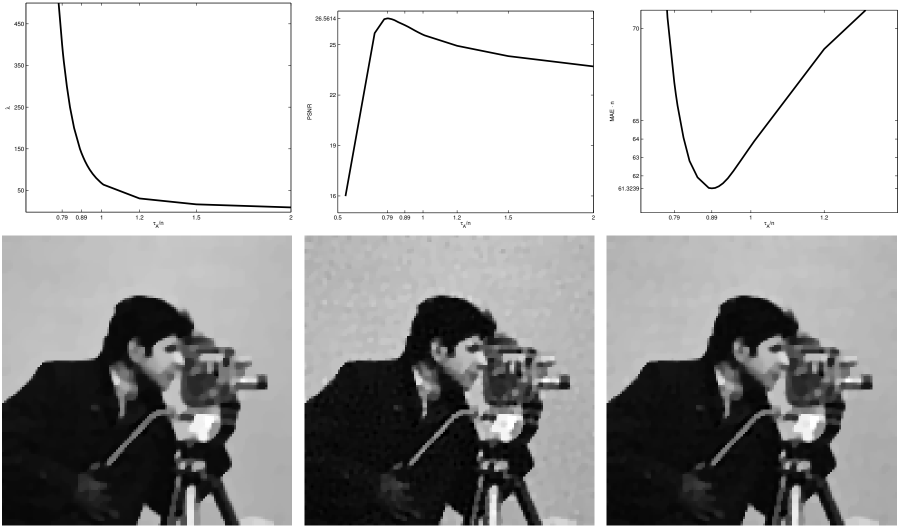

Figure : **Left:** u_n (PSNR=25.59, MAE = 63.62, $\tau = n$). **Center:** u_{PSNR} (PSNR=26.56, MAE = 67, $\tau_A = 0.792n$). **Right:** u_{MAE} (PSNR=26.16, MAE = 61.32, $\tau_A = 0.894n$).

Multi-constraint optimization

Block subdivision

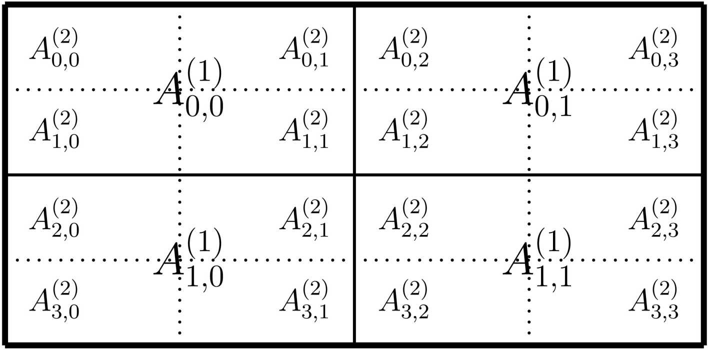

Figure : Domain partition for block subdivision of levels 1 (solid lines) and 2 (dotted lines).

Table : Results of Algorithms 1-2 on B1part$_{3000}$ for different levels l.

level	#iter	TV semi-norm	PSNR	MAE$\cdot\nu$
0	20000	1.7070e+6	25.5934	63.6238
		1.7073e+6	25.5949	63.6054
1	20000	1.7194e+6	25.6957	62.8892
		1.7197e+6	25.6975	62.8620
2	20000	1.7418e+6	25.8372	62.4555
		1.7423e+6	25.8385	62.4421
3	50000	1.7930e+6	25.9966	62.1405
	20000	1.7934e+6	25.9975	62.1315
4	50000	1.9490+6	26.0686	64.6669
	20000	1.9649e+6	26.0298	65.1237

Figure : Block subdivision. From left to right: Algorithm 1 outputs for $l = 3$ and $l = 4$.

Multi-constraint optimization

Intensity Tessellation

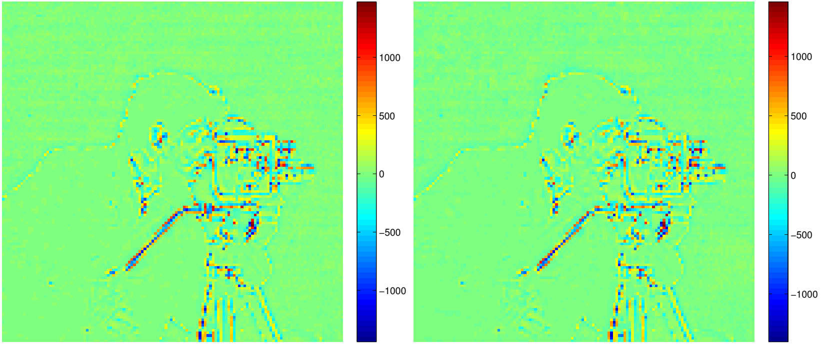

Figure : Noise redistribution in block subdivision. Left: $u_A^{(0)} - \bar{u}$. Right: $u_A^{(3)} - \bar{u}$.

$$A_i^{(l)} := \{j \mid \bar{u}_j \in (i2^{-l}3000, (i+1)2^{-l}3000]\}, \qquad \forall i = 0, \dots, 2^l - 1.$$

Table : Anscombe optimization for different levels of intensity tessellation.

level	#iter	TV semi-norm	PSNR	MAE$\cdot\nu$
1	20000	1.7080e+6	25.5810	63.6927
2	20000	1.7228e+6	25.7251	62.7592
3	20000	1.7439e+6	25.8739	61.4654
4	50000	1.7535e+6	25.8895	61.2834
5	50000	1.7695e+6	25.9421	61.5316
6	50000	1.7775e+6	25.9923	61.1921
7	50000	1.8046e+6	26.0401	61.0400
8	50000	1.8491e+6	26.1230	60.7815

Figure : The output images $u_A^{(4)}$ (left), $u_A^{(7)}$ (center), $u_A^{(8)}$ (right).

Table : Results of the 2-steps combined tessellation for different images and intensity levels. For all examples we set $(\sigma, \rho) = (0.4, 0.3)$, and $c = 9$.

Image	TV norm	PSNR	MAE$\cdot\nu$	TV$(u^{(1)})$	PSNR: $u^{(1)}/u^{(0)}$	MAE: $u^{(1)}/u^{(0)}$
B1part$_{3000}$	1.7573e+6	26.0000	60.9231	1.7194e+6	25.6957 / 25.5934	62.8892 / 63.6238
B1$_{100}$	1.2602e+5	24.4036	3.0603	1.2312e+5	24.3061 / 24.2844	3.0659 / 3.0761
B1$_{600}$	9.0592e+5	25.6788	15.2529	8.9419e+5	25.5992 / 25.5738	15.3439 / 15.4200
B1$_{1200}$	1.9653e+6	26.2329	28.6136	1.9339e+6	26.1012 / 26.0742	28.8040 / 29.0003
B1$_{2000}$	3.3570e+6	26.4457	46.1726	3.3308e+6	26.3738 / 26.3476	46.3732 / 46.6200
B1$_{3000}$	5.2662e+6	26.7679	66.7055	5.2092e+6	26.6491 / 26.6338	67.1725 / 67.5230
B2$_{100}$	9.9370e+4	19.9442	6.1998	9.3734e+4	19.8992 / 19.8949	5.9867 / 5.9804
B2$_{600}$	7.5496e+5	21.9439	23.7810	7.4231e+5	21.8235 / 21.8230	23.8747 / 23.8499
B2$_{1200}$	1.5802e+6	22.4779	42.5783	1.5561e+6	22.3595 / 22.3457	43.0012 / 43.0531
B2$_{2000}$	2.7236e+6	22.9163	64.9695	2.6747e+6	22.7625 / 22.7535	66.1393 / 66.1365
B2$_{3000}$	4.1120e+6	23.1652	93.2223	4.0658e+6	23.0049 / 22.9832	94.8958 / 94.9391

Multi-constraint optimization

2-step combined tessellation

Figure : Edge detection via difference images. Left: $u_A^{(1)} - f$. Center: $\bar{u} - f$. Right: $u_A^{(1)} - u_A^{(0)}$.